**DISCRETE** SCANNING **SYSTEMS**
FOR DIGITAL OPTICAL PROCESSING
AND TRANSFER OF IMAGES BY SYSTOLIC METHODS

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**We consider theoretically light beams propagating along the frontal rational trajectories within the nonparaxial geometrical optics region in the elements with reflecting cylindrical surfaces. We also discuss possible applications of the theory.**

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## 1. Introduction

Systolic processing matrices are used to construct the modern hardware systems [1,2]. Application of these processors using multilevel excess numerical systems {-k, 0, k} allows accelerated arithmetic, algebraic, and logic operations [3].

The systolic matrix implies a multidimensional architecture of parallel computers with the following peculiarities:

(i) each element of output data is simultaneously used to perform several calculations at a rather small volume of input-output operations;

(ii) a high processing speed is due to parallel employment of numerous simple processing elements (cells);

(iii) data and control connections between cells of the systolic matrix are simple and regular;

(iv) calculations are carried out similarly to heart muscle work, i.e., by pulse and simultaneously in the whole matrix volume [2].

In the present paper we propose the optical and optico-dectronic scanning devices corresponding to the above requirements and based on discrete effects in elements of cylindrical optics (mirrorlike or filled by a substance with refraction index higher than unity) beyond the paraxial optics region.

In contrast to the well-studied beam caustics producing a beam by cylindrical and semi-cylindrical lenses in a paraxial region, we consider the effects in discrete fans of beams outgoing from the regions of violated total internal reflection (vicinities of the edge points $C$ and $C'$ with coordinates $x = \text{A}$, $y = 0$, where $R$ is the surface curvature radius) of lens segments.

These phenomena were predicted in [3] where the beam trajectories were analyzed in a semi-cylindrical lens in the region of total internal reflection (RTIR) in terms of noncircular trigonometric functions. The effects were also experimentally observed in [4] and agreed with the theory of mathematical billiards [5]. The above approach uses the concept of nonparaxial optics.

For a cylindrical lens of radius $R$ (as well as for a mirror hollow cylinder),

$$X = R \cos \varphi, \; Y = R \sin \varphi,$$

which axis coincides with Z, there are longitudinal, transverse, and transverse-longitudinal geometric modes of light beams in the RTIR.

The beam propagation along polygonal trajectories of the transverse modes can be described by the current coordinates $X$ and $Y$ [4] (Fig. 1)

$$X = \frac{R\cos(\pi/k)}{\cos[\omega t - (2p+1)\pi/k - \varphi_0]}\cos\omega t, \quad (1)$$

$$Y = \frac{R\cos(\pi/k)}{\cos[\omega t - (2p+1)\pi/k - \varphi_0]}\sin s\omega t, \quad (2)$$

$p = 0, 1, 2, 3, \ldots, \quad \varphi_0 \in [0, \pi/k], \quad \omega t \in [2p\pi/k + \varphi_0; 2(p+1)\pi/k + \varphi_0]$

where $k$ = [2,3,...] is the fractal coefficient.

The lines of fc-polygonal traces of billiard trajectories can be revealed in the beam cross sections normally directed to the plane *ZX.* We note that in cylindrical elements within the non-paraxial geometrical optics region, $k$ can be arbitrary except for the values within the interval [-2,0,2]. The latter values of $k$ describe the beam trajectories in R-gradan lightguide systems, i.e., in selffocs.

If $k$ is integer, then, there are main transverse modes of billiard trajectories coinciding with regular $k$- polygons inscribed in a circle [5]. The trajectories are distinct for odd and even $k$, and we consider them below.

The way to input modes propagating along various $k$-polygonal trajectories was experimentally studied by us. A light beam enters the mirror hollow cylinder through the cylinder face or through holes and slots in surface. For the cylinder having n >1 or for segments of such a cylinder, the beam can enter the RTIR by the following four ways.

(1) The beam propagates tangentially to the cylindrical lens surface. Due to refraction and scattering by the microroughness of the cylindrical surface, a small portion of the beam energy enters the lens and propagates along the billiard trajectories.

(2) The light beam enters a local zone of the cylindrical surface with the help of a rectangular prism being in optical contact with the curvilinear surface or at distance A/2 from the surface, where \ is the beam wavelength. In this case, a marked portion of the beam gets into the RTIR.

(3) The directions of input beams should coincide with available billiard trajectories in angular sectors counted from the normales to the surface plane *ZX* of lens segments at the *C* and
*C'* points. The beams enter the RTIR from the external sides of the normales in the angular range from 0 to π/3.

The input angle ψ is related to the $k$ index of billiard trajectories by

$$\psi = \pi/k \quad (3)$$

(4) The beams directly enter the RTIR along the normal or at angles $\pm\pi/2-\varphi_{in}$ to the *ZX* plane of lens segment restricted by $X = R$ and

$$X_{in} = R\, Sin\, \varphi_{in} = Rn_1/n_2, \quad (4)$$

where *y\n* is the angle of the RTIR beginning and sin *y\n* = Hi/Ha are the ratio of refraction indices in air and the lens material.

In this case, 96-98% of total energy can enter the RTIR. The position of the input point $X_{in}$ (when the beam is injected in the normal direction to the *ZX* plane) is related to the $k$ factor by

$$X_{in} = R\, Cos\, (\pi/k), \quad (5)$$

We note the $k$ index characterizes the radius r of the lens region where beams with multiplicity higher than $k$ do not propagate for all the billiard trajectories,

$$r = R\ Cos\ (\pi/k), \tag{6}$$

If $k$ is described by a rational fraction

$$k = n/m, \tag{7}$$

then, the beams propagate along closed starlike trajectories having $n$-nodes formed by the beam rotating m times in a circle with radius $R$ or in a ring with radii $R$ and $r$. Such $k$-polygons have a fractional dimension, i.e., they are fractal $k$-polygons.

Figure 1 plots the fractal $k$-polygonal trajectories in a circle when the injected beam is directed normally to the axis $X$ (for $k$ from 3 to 10/3). There are trajectories passing through the points $C$ and $C'$ as the beam rotates in circle. For instance, if $k = n/m$, the beam starting from the node $p = (n+ 1)/2$ passes through the point $C$ (if the beam is injected into RTIR

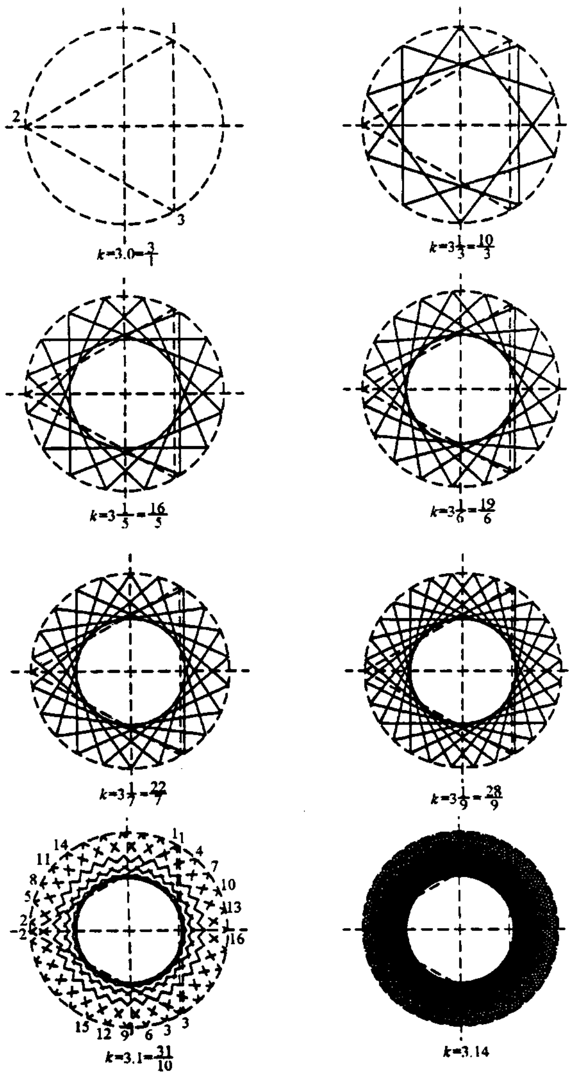


**Figure 1.** Trajectories of light beams propagating as transverse modes ($k$ varies from 3 to 10/3).

along the normal to the plane $ZX$ by the fourth method). For beams outgoing from a segment, the face between the plane $ZX$ and the cylinder surface is a mirror shield, which restricts periodically the angular input sector as $k$ varies continuously. As a result, we observe a set of beam fans with the monotonically decreasing divergence angle. The beam angle with respect to the normal to the output plane of a cylindrical segment is related to the divergence angle $\gamma$ as

$$a = \frac{2\pi}{k}\left(\frac{1}{2} - \left\{\frac{\gamma k}{2\pi} - \frac{1}{2}\right\}\right),$$

where we imply that the segment has the axis coinciding to the axis *Z* and is restricted by the planes *ZX* and *ZX.* In the above expression { } is the fractional part of the number, while $X_1$ lies at angle $\gamma$ with respect to *X.*

The angular distribution of light flux in the output plane $ZX_1$ depends on interaction of waves forming the front of incoming wave, on properties of cylindrical element, and on diffraction effects at the segment output face. This distribution is discrete, and its main characteristics are the geometrical characteristics of beams propagating along rational trajectories closed at the points *C* and *C'.*

The similar scanning structures can be formed in the distorted RTIR in a hollow refracting cylinder or in a mirror hollow cylinder with the slot output.

Thus, scanning beams outgoing in the vicinity of the points *C* and *C'* at angles from 0 to $\pi/k_{min}$ where $k_{min} = 2\pi/(\pi - 2\varphi_{in})$ are counted from the normal to the surface *ZX.* As well as in the case of a semicylindrical lens, these beams include the ones propagating along irrational or transcendent *k*-polygonal trajectories and rotating many times along the circle, totally filling it by nodes, and never passing through the points *C* and *C'* (the Jacobi theorem) [5] (see Fig. 1 for *k* =3.14...).

These peculiarities of angular scan formation for the beams propagating in mirror cylindrical elements result directly from (1) and (2), applied to both the direct and backward propagations of the beams. In these expressions to describe the beams outgoing from the vicinities of the points *C* and C', the initial phase yo is assumed to be zero. The energy of the beams outgoing from the points *C* and $C^9$ is approximately 30% of the total input energy.

It is noteworthy that due to the mirror surface *ZX* the semicylindrical lens transforms the most part of beams of the first and second quadrants into the ones of the third and fourth quadrants of circle. Thus, the semicylindrical lens with additional mirror coating is in essence an optical detector in the geometrical optics region, i.e., it transforms the operations in the excess {*-k, 0, k*} numerical system into those in the *k*-fold numerical system.

In addition to the above methods, the longitudinal modes have an angular displacement of the input beam along the axis *Z* of the cylinder. In this case the transverse modes of partial beams have a spiral displacement along the cylinder axis, and the beams forming *k*-polygon do not lie in a plane, but they coincide with faces of generating prism (Fig. 2) generally to be starlike and fractal. The integer ratio between the length of a smooth screw line connecting the points of reflection for end and beginning of the screw turn and the circle length in the normal cross section of cylinder corresponds to the number of longitudinal modes for which the transverse modes have

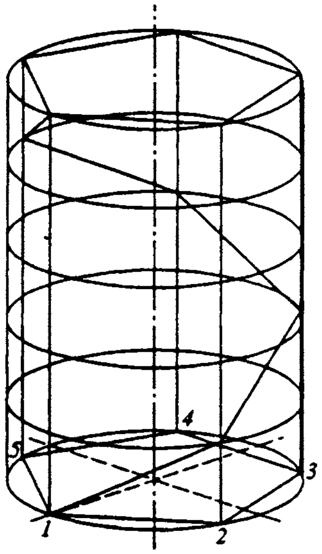


**Figure** *2.* Trajectory of transverse-longitudinal mode at $k=5$ and $\beta=arctan(4/3)$.

the transverse index $k > 2$. We refer these modes to as transverse-longitudinal ones. They propagate in accordance to (1) and (2), with regard to the corresponding index / of transverse modes and the trigonometric dependence on screw angle. The current coordinates $Z_{k,l}$ for $k$-polygons are determined by

$$Z_{k,l} = 2pR \cdot \cos(\pi/k)\tan\beta = 2plR \cdot \cos(\pi/k)\sin\beta \text{ ,(8)}$$

taking into account the relation $l = 1/\cos\beta$.

The total length of this screw-segment trajectory to the node p is equal to

$$L_{k,p} = 2pR\frac{\sin(\pi/k)}{\cos\beta} = 2plR \cdot \sin(\pi/k) \text{ ,(9)}$$

where $p$ is an integer positive number. If $p = n$, the screw-segment trajectory makes a single turn along the cylinder generatrix. For partial beams at a chosen point on the cylinder generatrix passing through the input point $C$, the temporal delay is equal to $t = L_{kn}/c$ where $c$ is the light velocity and $p = n = km$.

Index $l$ of transverse-longitudinal modes as well as index $k$ of transverse modes can be arbitrary in general. In such a cylinder the transverse-longitudinal modes propagate along right and left screw-segment trajectories determined by (8) and (9), depending on sign of the angle. The central longitudinal mode is observed at $\underline{k} = 2$ and $l \to \propto$.

The described effects of controlled scanning beams and transverse and longitudinal propagation of light in optical mirror cylindrical elements can be applied to construct new optical and optico-electronic devices of unique properties. We consider below some such devices.

## 2. Optical processing by systolic methods

As was shown in [3], spatiotemporal discretization of (1) and (2) can be used to produce symbolic digital discrete trigonometric functions realizing integer sequences in positional numerical systems. The author of [3] also derived equations of direct and inverse spectral expansions, which were additive discrete analogs of the Fourier transform.

The derived transform can be applied for any numbers and uses, as an orthogonal set, the positive and negative branches of Loran series with alternate sign within the intervals *n* of digits from

$a_{-m}k^{-m}$ to $a_nk^n$. As a result, the optical systems calculating these spectral expansions can be presented as a semicylindrical lens for one-dimensional sequences or a pair of semi-cylindrical and torus-cylindrical lens for the two-dimensional ones.

In the future, recurrent methods of automated analysis, compression, and synthesis of systolic structures can be realized to combine transverse modes of a semicylindrical lens for two or more coherent sources of modulated radiation [6]. These suggestions are based on self-synchronization of the modes and determined by the geometrical sizes and shape of the optical cylindrical elements.

We should emphasize additional abilities for constructing the commutation systems to transfer organized optical data by using decentral elements with the second order nonconcentric surfaces. The cadence frequency for operations with arrays of systolic data is determined by the time of light flux transfer along the modes with the fixed *k* index.

## 3. Fiber optics

The input of optical radiation is well known to be too difficult for the fiber-optics systems. Light is usually introduced into the carrying fiberguide through its face, but effectiveness of the input is rather small. If it is necessary to input an intense beam or to inject it from the sources with distinct spectrum, they use the compression systems based on diffraction elements [7,8].

If fiberguide face is made as an semicylindrical polished surface with linear cross section restricted by the semicylinder generatrix and counted with respect to the guide axis, $X_{in}=R(n_1/n_2)$, where $n_1$ is the refraction index of the fiber and $n_2$ is that index of the fiber coating, then, a sufficient number of radiation sources including those with different spectrum can be used in the fourth input method. In this case we should make windows on the plane surface by reflective coatings on free zones of the RTIR. If the beam is directed at an angle to the fiber axis *Z*, we can excite longitudinal modes propagating with a fixed velocity along screw billiard trajectories. Radiation propagates by transverse-longitudinal modes and by central longitudinal one.

Effectiveness of the above method was proved by us experimentally. We note that this method can be used to construct new fiber pickups using counterpropagating beams (for instance gyroscopic ones), since the semicylindrical surface can be formed at any place of the fiber.

## 4. Systems of laser illuminating and ranging with optical and spatial modulation

Using various methods of the beam input and choosing various forms of optical cylindrical elements and substances filling them, we can by theoretical and experimental methods construct the systems of one- and two-dimensional illumination in a wide sector of divergence angle from 10-15 to 360° for a one-dimensional case and from solid angle 10x10° to a hemisphere for two-dimensional continuous and scanning illumination. The illumination methods using the above elements provide the transverse motion of the beam to be transformed in sector-angular and circular displacement.

## 5. Control of self-synchronous modes in waveguide ring resonators

Active systems can be based on optical elements, where transverse, transverse-longitudinal, and longitudinal modes exist as circulating light beams, self-synchronized and tightly connected with geometry and properties of the active medium. At the trajectories of self-synchronized waveguide modes, the free path is $L_{k,n}=q\lambda/2$, where $q$ are integers [1,2,3,...Q]. In such systems, high-quality resonators, the continuous self-synchronized modes can be generated as well as the pulses of discrete frequencies. We suggest to construct the systems with external and internal elements to control the resonator quality.

An internal control of the quality can be carried out by the modulation of distribution of the reflection, scattering, and absorption coefficients as well as of polarization parameters, and by formation of local decentral second order surfaces of resonators. Theoretical study of interaction of transverse and transverse-longitudinal modes with a surface segment formed by the cylinder crossing the plane parallel to the cylinder axis show possible commutation of modes in the resonator separately or in combination.

In the torus-cylindrical resonators, the generation of transverse-longitudinal modes can be controlled by the choice of ratio between external and internal diameters in the plane *ZX.* The problems of radiation input and choice of modes is solved, for instance, by local distortion of total internal refraction with the use of programmed light detectors controlled by piezoceramic and electrooptic pickups.

**6. Conclusion**

In this paper **we** generalize our previous studies devoted to the theory and application of scanning and caustic effects for optical beahis propagating along billiard trajectories in mirror cylindrical elements. Multilevel synthesis of such elements together with the treatment of beam input-output setups can be widely used to apply these elements for self-synchronization and spatiotemporal coupling between transverse and longitudinal modes of the billiard trajectories. The elements can be also useful for construction of optical instruments in the fields:
parallel data processing with the use of new mathematical formalism, fiber-optics technology, image processing and transfer, resonator systems with controlled quality. The common feature of these lines is due to unique possibilities of the RTIR optics to control the apparatus functions, to form and commutate dense data with a high speed comparable to the light velocity.

**References**


1. KungH.T. Computer 1982, 15(1), 37.

2. Kukharev G.A., TropchenkoA.Yu., and ShmerkoV.P. Systolic Processors for Signal Processing. Minsk: Belarus, 1988 [in Russian].

3. Melnikov G.S. Digital Signal Processing by Systolic Processors. Preprint 9-91. Lvov: Integral, 1991 [in Russian].

4. Melnikov G.S., Kosmachev A.F., and Shishkin M.Yu. Proc. 4th All-Union Conf. on Theoretical and Applied, Optics. Leningrad. State Optical Institute, 1986 [in Russian].

5. Galperin G.A. and Zemlyakov A.N. Mathematical Billiards. Moscow: Nauka, 1990 [in Russian].

6. Melnikov G.S. Proc. 1st All-Union Conf. on Pattern Recognition and Analysis. Pt. 2. Minsk, 1991, p. 18 [in Russian].

7. MikheevP.A. *Opt. Mekh. Prom.* 1982, No. 8, 13 *(Sov.* 7. *Opt. TechnoL).*

8. Tarasov L.V. Physics of Processes in Generators of Coherent Optical Emission. Moscow: Radio i Svyaz, 1981 [in Russian].