FRACTAL OPTICS

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The paper deals with the theory and experiment of fractal optics. Described are results of experimental tests of fractal properties of photon paths propagating in light conducting cavities of cylindrical type along modal paths (geometric longitudinal, transversal, longitudinal-transversal and longitudinal-radial modes). Basing upon analysis of these properties and attracting results of development of the theory of numbers (integers, rational, irrational) the author suggests his interpretation of gnoseology of fractality [1]. Discussed are equations of movement of photons in light conductors. Properties of new optical elements, multimode multilevel Fabry-Perot interferometers, are considered.

1. THEORY OF PROPAGATION OF GEOMETRIC RAYS IN ELEMENTS OF FRACTAL OPTICS

Consider some basic fundamental tenets of propagation of light rays in cylindrical bodies similar to the principles worked out for description of laws of mathematical billiards.

As mentioned above the polygons forming n apexes (points of reflection during m "rotations" around a reflecting surface curvature centre, or a curve in two-dimensional consideration), i.e. polygons having a fractality coefficient of

$$k=n/m \qquad (1)$$

can be named fractal polygons.

If in the general case the fractality coefficient is presented as k = n/m, and k is fractionary-rational, we have a polygon closed at n-th reflection. In this case a polygon side (in a light conducting element this is a free path from one reflection to another) runs along a finite area during m rotations. In case if k is transcedent, movement of apexes of such a polygon and running along a finite area takes place, when n and m tend to infinity. In so doing the current apexes never coincide with the first reflection point. In other words the transcedent fractal polygon is an unclosed figure.

1.1. THEORETICAL DESCRIPTION OF PATHS OF PARTIAL LIGHT RAYS IN SINGLE-FRACTAL CYLINDRICAL LIGHT CONDUCTORS

We derived formulae permitting to structurize propagation of partial light rays in the mode of multiple reflections from curvilinear and plane surfaces restricting optical light conducting elements of fractal optics basing upon the above physical and theoretical prerequisites [1]. As is found an initial homocentric light beam in the first reflection is divided into partial groups characterized by rational values of fractality coefficients. These groups correspond to transversal, longitudinal and longitudinally-transversal geometric modes and submodes.

In such a way the author managed to eliminate uncertainty principle in problems of description of light ray current coordinates (current position of a single photon moving in a light conducting element in the process of multiple reflection from interfaces restricting the light conductor).

Therefore formulae of non-circular trigonometry [1] suitable for determination of coordinates X,Y (in plane Z = 0) can be used with success for description of transversal geometric modes of single-fractal optical elements of circular cylinder. These expressions are derived by the author in work [2].

$$\sin_{k,\varphi}(\omega t) = \frac{\cos\left(\frac{\pi}{k}\right)\cdot\sin(\omega t)}{\cos\left(\omega t - \frac{\pi(2p+1)}{k} - \varphi_0\right)}, \quad \textbf{(2)}$$

$$\cos_{k,\varphi}(\omega t) = \frac{\cos\left(\frac{\pi}{k}\right)\cdot\cos(\omega t)}{\cos\left(\omega t - \frac{\pi(2p+1)}{k} - \varphi_0\right)} \quad \textbf{(3)}$$

with the given initial and boundary conditions: $\mathbf{0 \le \varphi_0 \le \pi/k,\ p = 0,1,2,...;\ \ \omega t \in [2\pi \mathit{p}/k + \varphi_0;\ 2\pi(\mathit{p}+1)/\mathit{k} + \varphi_0];\ \mathit{k} \ge 2.}$

For description of longitudinally-transversal modes one should add parameter l connected with inclination β in plane XY. In this case Z can be determined by the formula

$$Z_{k,l} = 2\tau R\cos(\alpha)tg(\beta) = 2\tau l r\cos(\alpha)\sin(\beta) = 2\tau l R\sin(\pi/k)\sin(\beta) \qquad (4)$$

where $l = 1/\cos(\beta)$, $k = n/m$, $\tau \in 0,1,2.3...$

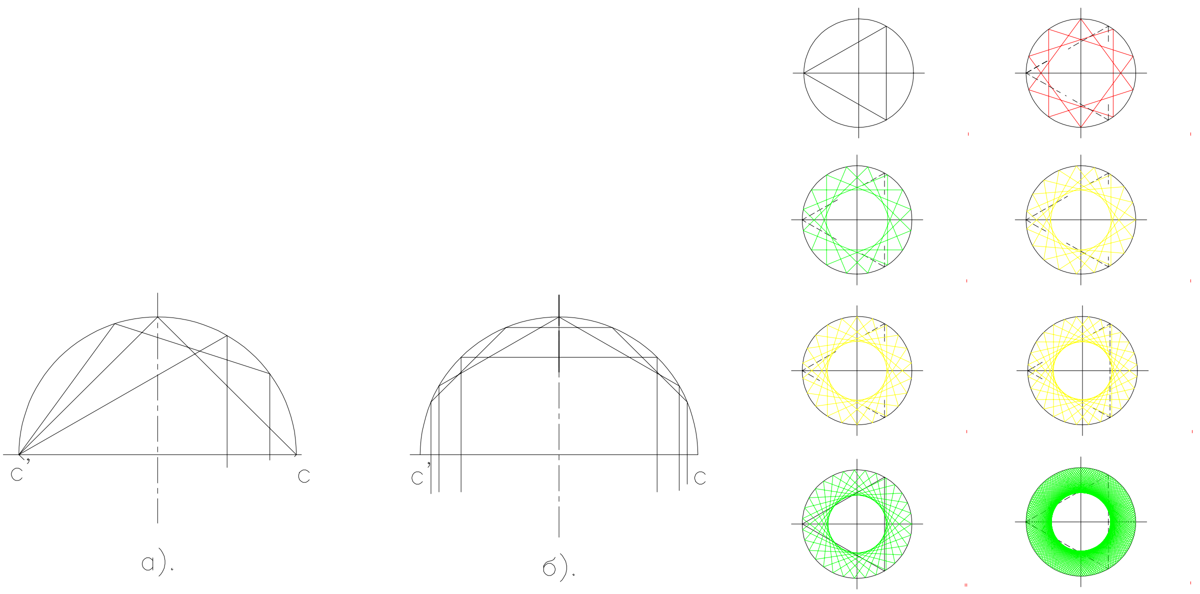


Fig. 1. Results of mathematical simulation of photon paths in single-fractal light-conducting systems

Results of mathematical simulation of photon paths in single-fractal light-conducting systems are shown in Figs. 1a and 1b.

1.2. THEORETICAL DESCRIPTION OF PATHS OF PARTIAL LIGHT RAYS IN BIFRACTAL CYLINDRICAL LIGHT CONDUCTORS

Procedure of propagation of light rays in bifractal light conductors (tubular cylinders) gets complicated due to the fact that extraparaxial light rays undergo multiple reflections not only from the external curvilinear surface of radius R restricting the element, but they can be reflected also from the internal surface of radius r.

In case when there is no reflection of rays from the internal surface, or rays are reflected tangentially to the interface of radius r, the light rays are described by conventional expressions given above for single-fractal light conducting systems characterizing transversal and longitudinally-transversal modal paths.

When rays are reflected in turn from surfaces of radius **R** and **r**, we have the general case of description of movement along radially-transversal modal paths.

For derivation of general equations of ray propagation we consider example of rational division of circles of radius **R** and **r** by a finite amount of points of light ray reflection. In other words for derivation of general expressions specifying position of transversal and radially-transversal modal paths we proceed from consideration of simplified case, i.e. from the case when the **R-r** relationship and fractality coefficients **k = n/m** and **p =** η/μ determine divisibilities of these circles as integer and fractionary- rational ratios. The given case is shown in Fig. 1b. (**k = 8; p = 3**).

From Fig. 2 one can see that a light ray entered from point **M** in the light conducting cavity restricted by interfaces of radii **R** and **r** at angle Ψ to normal **OM** passing through the centre of above concentric circles is in turn reflected from surfaces of radii **r** and **R** at angles φ and Ψ, respectively, from similar normals to the internal and external restricting surfaces.

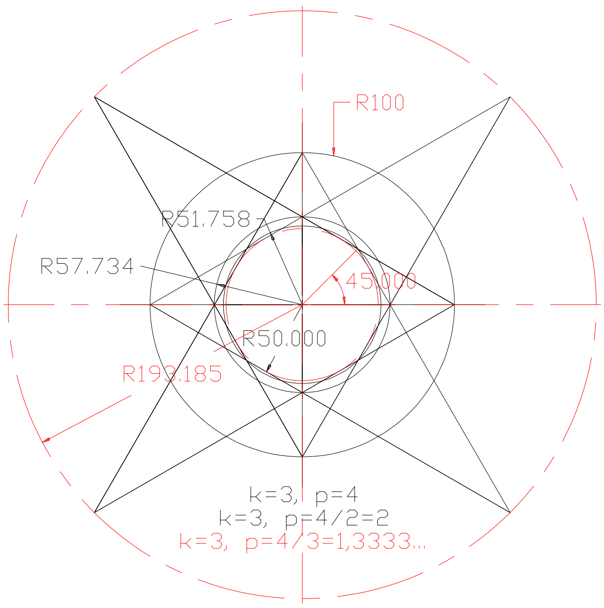


Fig. 2. Results of mathematical simulation of photon paths in bi-fractal light-conducting systems

As a result of consideration in plane **X,Y** of a system of curvilinear triangles of **OMN** and **NOQ** type one can write the following basic relationships:

$$\gamma = 2\cdot\pi/\rho(\mu,\eta)\ ;\alpha = \pi/2 - \pi/\kappa(m,n)$$

$$\text{и } L/\sin(\pi/\rho(\mu,\eta))=r/\sin(\pi/2 - \pi/\kappa(m,n))=R/\sin(\pi/2 -(\pi/\rho(\mu,\eta)-\pi/\kappa(m,n)) \quad \textbf{(5)}$$

Basing upon them we suppose, that in its current movement a partial group of rays (photon trains) carries out current movement within a ring of concentric circles **R** and **r** with periodical change of movement direction at boundaries of curvilinear restricting surfaces.

In this case we may introduce an angular parametric function

$$G(m,n,\mu,\eta)=\cos(\pi/k)\cdot\cos(\pi/p)/(\cos(\pi/k)-\sin(\pi/p)\cdot-\sin(\pi\cdot(k-p)/k\cdot p)) \quad (6)$$

connected with fractality coefficients.

For the circle of radius **R**:

$$\kappa(m,n)=n/m \quad (7)$$

For the circle of radius **r**:

$$\rho(\mu,\eta)=\eta/\mu \quad (8)$$

Commutation function at reflecting surfaces (function of current time is expressed in current change of reflection points)

$$S(\tau)=(1+(-1)^{\tau})/2 \quad (9)$$

and as a result function of current movement

$$F(m,n,\mu,\eta,\tau)=G(m,n,\mu,\eta)+S(\tau)\cdot G(m,n,\mu,\eta) \quad (10)$$

whence for current coordinates X and Y we obtain expressions

$$Y(m,n,\mu,\eta,\tau)=R\cdot F(m,n,\mu,\eta,\tau)\cdot\sin(\tau\cdot\pi/p) \quad (11)$$

$$X(m,n,\mu,\eta,\tau)=R\cdot F(m,n,\mu,\eta,\tau)\cdot\cos(\tau\cdot\pi/p) \quad (12)$$

As seen the commutation function changes from 0 to 1.

When $S(\tau)=0$ photons are reflected from the surface with radius **r**

$$r=R\cdot G(m,n,\mu,\eta) \quad (13)$$

When commutation function $S(\tau)=1$, photons are reflected from the surface of radius R. When τ has intermediate values, photons in light conductors of material with the same refraction index move along a straight line between two reflection points corresponding to

$S(\tau)=0$ and $S(\tau)=1$.

For description of movement of photons along radially-longitudinal mode paths in addition to expressions **(11)** and **(12)** the current values of coordinate **Z** are determined by expression **(4)**.

From analysis of expressions **(4 ... 12)** one can see that in bifractal systems fractality coefficient $\kappa(m,n)=n/m$ determines divisibility of restricting surfaces of radii **R** and **r** expressed as integer, rational, irrational and transcendent ratios.

Fractality coefficient $\rho(\mu,\eta)=\eta/\mu$ connected with sectoral-angular division of light conducting ring of bifractal cylindrical optical systems and is specified by an angle of entrance in a light conducting cavity of rays in point 0. (Fig. 1, and expression **(5)**).

In this case:

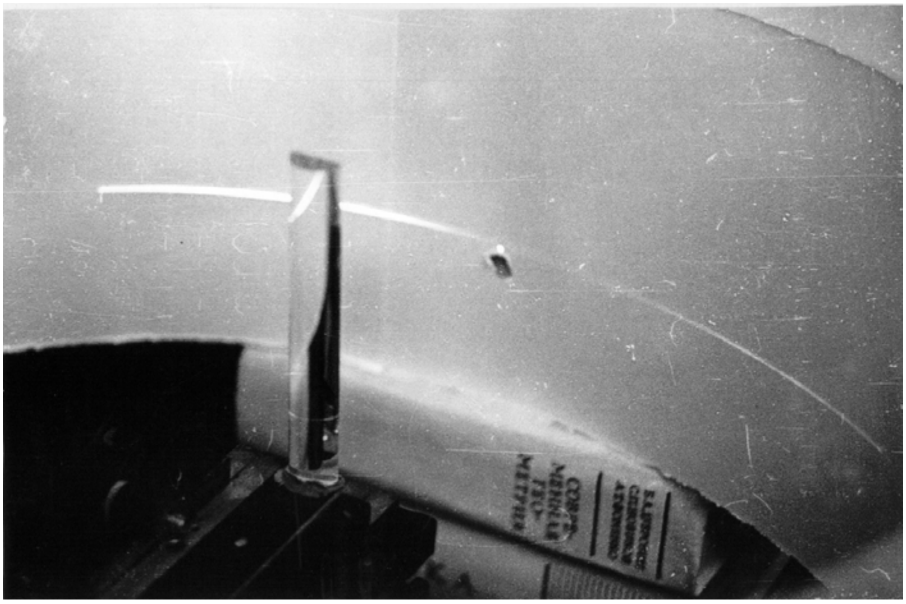
When fractality coefficients are equal $\kappa(m,n)=\rho(\mu,\eta)$ expressions **(11)** and **(12)** transforms into expressions **(2)** and **(3)** and small circle transforms into a boundary concentric circle, whose radius is determined by expression **(13)**, reflecting light rays tangentially to circle **r**, which proves in such a way that the bifractal light conducting system for which $\kappa(m,n)\geq \kappa(m,n)=\rho(\mu,\eta)$ in this case is considered as a single fractal system.

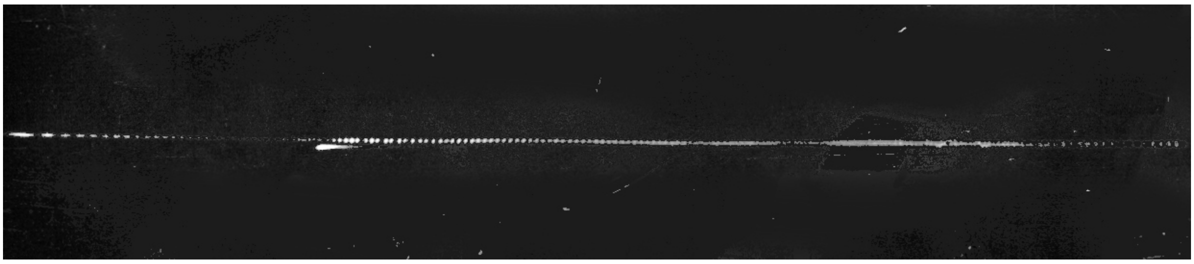
From **(4 ... 13)** one can also see that for creation of the light conducting system with rational fractality coefficients $\kappa(m,n)$ and $\rho(\mu,\eta)$ internal radius **r** of tubular cylinder should be (for each radially-transversal mode) of a certain value specified by expression **(13)**.

For arbitrarily selected bifractal systems the R - r relationship characterizes those radially-transversal modal and submodal paths, that for the given external and internal radii can close up at m-th rotation in accordance with expression **(8)**.

Taking into account, that at a certain relationships between radii and entrance angles α close but not equal to the critical angle for total reflection, in each reflection point (τ_i) there can appear such conditions, when a part of entered light beam energy goes out of the light conducting cavity creating a raster of rays propagating within an angle sector of up 360^0.

All the above theoretical prerequisites have been proved in experiment.





On Fig. 3 shows photos of experimental observation of geometrical modal paths by means of elements of fractal optics.

It is typical that angular positions of partial rays filling the raster are in full conformity with expressions **(2), (3), (4)** for single-fractal optical elements and expressions **(11), (12), (4)** for bifractal elements.

The author obtained also results concerning description of photon paths by equations of macrowave amplitude-angular modulation of form:

$$Y=R_0+U_{m0}(\varphi)\cdot S'(t)\cdot \sin(\omega t); \qquad (14)$$

$$X=R_0+U_{m0}(\varphi)\cdot S'(t)\cdot \cos(\omega t); \qquad (15)$$

In these expressions amplitude and angular modulating functions **S'(t)** and ($U_{m0}(\varphi)$), respectively, have frequency of change exceeding frequency of carrier ω. Their periods are connected with free time of photons from one reflection from the surface of radius **r** to another reflection from the surface of radius **R.**

The fact that geometrical modal paths of photons propagating inside an element of fractal optics are described by macrowave equations provides a unique possibility to construct a number of new devices of discrete digital optics.

However, obtained body of mathematics is beyond the scope of the given paper and is a subject of independent research work.

Results of the mentioned studies will be published within the planned series of papers.

CONCLUSIONS

1. Fractal properties of a new class of extraparaxial multiple reflection optical elements are shown.

2. Mathematical simulation confirming multiparametrical fractality of movement of light rays in decentred light conductors is carried out.

3. Developed are elements of mathematical model of propagation and transformation of light rays and wavefronts for centred elements of fractal optics.

4. It is shown that it is possible to overcome increase of dimensionality of parametric space of light ray propagation in different classes of light conducting elements from the point of view of their theoretical description.

In so doing some special properties of fractal optics has been revealed:
- hierarchic structure of propagation of ray caustics in light conductors,
- possibility to use principles of self-similarity of ray paths for creation of digital illumination systems,

- hierarchic distribution of entrance points and angles of rays in segmented elements of fractal optics providing possibility to create the scanning systems with optical reduction of sectors of caustic and raster rays of formed coherent and nonmonochromatic radiation depending on certain requirements.

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